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Electromagnetic Field at the Transition Zone Between two Different  
Sections of Earth's Surface

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The purpose of present research was to investigate in a possibly rigorous way the field in the neighbourhood of a transition zone between two different sections of earth's surface in order to find the magnitude of field disturbance created at the boundary and to verify the formulae of conventional inhomogeneous earth theories.

The formulation of the problem is as follows:

The  $x, y$ -plane is the plane of earth's surface. The  $z$ -axis is directed upwards towards the homogeneous atmosphere. To the left of the line  $x = 0$  (zone "a" - Fig. 1) the earth is homogeneous with constant surface impedance  $Z_s = Z_a$ ; to the right of the line  $x = d$  (zone "b") the earth shows constant surface impedance  $Z_s = Z_b$ . In the transition zone (zone "t") of the width  $d$  the surface impedance  $Z_s$  changes linearly from  $Z_a$  to  $Z_b$ . This is an essential assumption since Wait investigating this problem previously (1957) under the assumption of a sharp discontinuity in surface impedance found a disquieting field singularity at the boundary. The transmitting aerial (aerial 1) is a short vertical dipole situated just over the earth's surface at the point  $A_1$  at a large distance from the boundary. The electric moment of the dipole is  $p$ ; the field produced by it will be denoted by  $\bar{E}_1$ ,  $\bar{H}_1$ . In view of a large distance from the aerial 1 to the boundary the wavefronts over the zones "b", "t" and also over the zone "a" in the neighbourhood of the boundary will be assumed to be parallel to the  $y$ -axis.

Next we introduce a second, auxiliary and fictitious problem: the same atmosphere as before, a plane homogeneous earth of surface impedance  $Z_a$ , and the transmitting aerial (aerial 2) in form of a

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short vertical dipole of electric moment  $p$  situated just over the earth's surface at the point  $A_2$ . This auxiliary fictitious field will be denoted by  $\bar{E}_2$ ,  $\bar{H}_2$ .

The problem will be solved by means of a vector integral equation derived by the author previously (Godzinski, 1961) which for the present purpose may be put in the form

$$E_{1v} = E_{2v} - \frac{i}{\omega p} \int_S (Z_a - Z_s) (\bar{H}_{1t} \cdot \bar{H}_{2t}) dS \quad (1)$$

In this equation  $E_{1v}$ ,  $E_{2v}$  denote the vertical components of electric field intensities produced by aerial 1 at the reception point  $A_2$  over the real and the auxiliary homogeneous earth, respectively.  $\bar{H}_{1t}$ ,  $\bar{H}_{2t}$  denote the tangential components of magnetic field intensities,  $\omega$  is the angular frequency, and  $Z_s$  is the surface impedance of the real earth. The surface integral extends in principle over the whole surface  $S$  of the earth. As, however,  $Z_s = Z_a$  over the zone "a", the surface integral extends practically only over the zones "t" and "b".

When calculating this integral we may divide the surface of integration into narrow stripes and then subdivide the stripes into elements  $dS = dy d\xi$  (Fig.1). The contribution from the element  $dS$  is equal to  $-\frac{i}{\omega p} (Z_a - Z_s) \bar{H}_{1t} \cdot \bar{H}_{2t} dS$ . As  $\bar{H}_{1t}$  is parallel to the  $y$ -axis, the  $y$ -component of  $\bar{H}_{2t}$  needs only be taken into account. Calculating  $\bar{H}_{2t}$  we may interchange formally the transmitting and receiving points. Accordingly, we will consider formally aerial 2 to be placed not at the point  $A_2$  but at the point B (Fig. 1), and then will compute the field it produces at the point  $A_2$ . Adding up the contributions from all elements  $dy d\xi$  of a stripe it is easy to show that the resultant magnetic field is the same as that obtained from a uniform distribution of vertical dipoles along the stripe, i.e. the same as that produced by a double line source consisting of vertical electric moments, parallel to the  $y$ -axis and placed just over the surface of the stripe; the electric moment per unit length of this double line source is proportional to  $H_{1t}$ .

When calculating the magnetic field of such double line source it is in principle necessary to take into account the properties of the auxiliary homogeneous earth (of surface impedance  $Z_a$ ) over which the field is generated and to derive an expression for the suitable attenu-

ation function. However, as the purpose of the investigation is to find the field in the neighbourhood of the transition zone only the problem may be simplified considerably by the assumption that at the considered short distances the ground influence is negligible. For the present purpose we may thus replace the ground by a perfectly conducting plane; the field is then twice that for free space propagation. In order to calculate this field we introduce the vertical Hertzian vector  $\vec{\Pi}$  and determine its two-dimensional Green's function. The magnetic field produced by the stripe we then calculate readily as  $i\omega\epsilon_0\text{curl}\vec{\Pi}$ . The total field generated by all secondary sources distributed over the earth's surface will be obtained by adding up the contributions from all elementary stripes; eqn. (1) reduces thus to

$$E_{1v} = E_{2v} + \frac{i}{2} \gamma_0 \int_0^\infty H_{1t}(Z_s - Z_a) \frac{x - \xi}{|x - \xi|} H_1^{(2)}(\gamma_0 |x - \xi|) d\xi \quad (2)$$

where  $H_{1t}$  denotes the unknown tangential component of magnetic field over the inhomogeneous earth, and  $H_1^{(2)}$  is the Hankel function of the second kind;  $\gamma_0$  is the free space propagation coefficient.

Thus the two-dimensional integral equation (1) has been reduced to a much simpler one-dimensional integral equation (2). The only approximation consisted in omitting the attenuation function; by taking it into account the transformation could be made strictly rigorous. Thus all errors connected with the conventional application of stationary phase principle have been avoided and a rigorous method has been found which may be of use in a number of similar problems.

If  $E_{1v}$ ,  $E_{2v}$  and  $H_{1t}$  are expressed by their attenuation functions eqn. (2) reduces after obvious approximations to the final form

$$\frac{w_1(r_0 + x)}{w_a(r_0)} - 1 \approx -\frac{i}{2} \zeta_b W(\zeta) \quad (3)$$

where  $w_1(r_0 + x)$ ,  $w_a(r_0)$  - attenuation functions over the real earth at the distance  $r_0 + x$  from the transmitter, and over the auxiliary homogeneous earth of surface impedance  $Z_a$  at the distance  $r_0$  from the transmitter, respectively

$$\zeta_b = (Z_b - Z_a)/Z_0$$

$$Z_0 = (\mu_0/\epsilon_0)^{\frac{1}{2}} - \text{intrinsic impedance of free space}$$

The function  $W(\zeta)$  is equal

(a) for the zone "a" ( $x \leq 0$ ;  $\zeta \leq 0$ )

$$W(\zeta) = \frac{1}{\delta} [B_r(-\zeta + \delta) - B_r(-\zeta) - (-\zeta + \delta) A_r(-\zeta + \delta) - \zeta A_r(-\zeta)] \quad (4)$$

(b) for the zone "t" ( $0 \leq x \leq d$ ;  $0 \leq \zeta \leq \delta$ )

$$W(\zeta) = \frac{1}{\delta} [B_r(\delta - \zeta) + B_t(\zeta) - (\delta - \zeta) A_r(\delta - \zeta) - \zeta A_t(\zeta)] \quad (5)$$

(c) for the zone "b" ( $d \leq x$ ;  $\delta \leq \zeta$ )

$$W(\zeta) = \frac{1}{\delta} [B_t(\zeta) - B_t(\zeta - \delta) - \zeta A_t(\zeta) + (\zeta - \delta) A_t(\zeta - \delta)] \quad (6)$$

where  $\zeta = \gamma_0 x = 2\pi x / \lambda_0$ ;  $\delta = \gamma_0 d = 2\pi d / \lambda_0$ .

The four functions  $A_r$ ,  $A_t$ ,  $B_r$ ,  $B_t$  are defined as follows

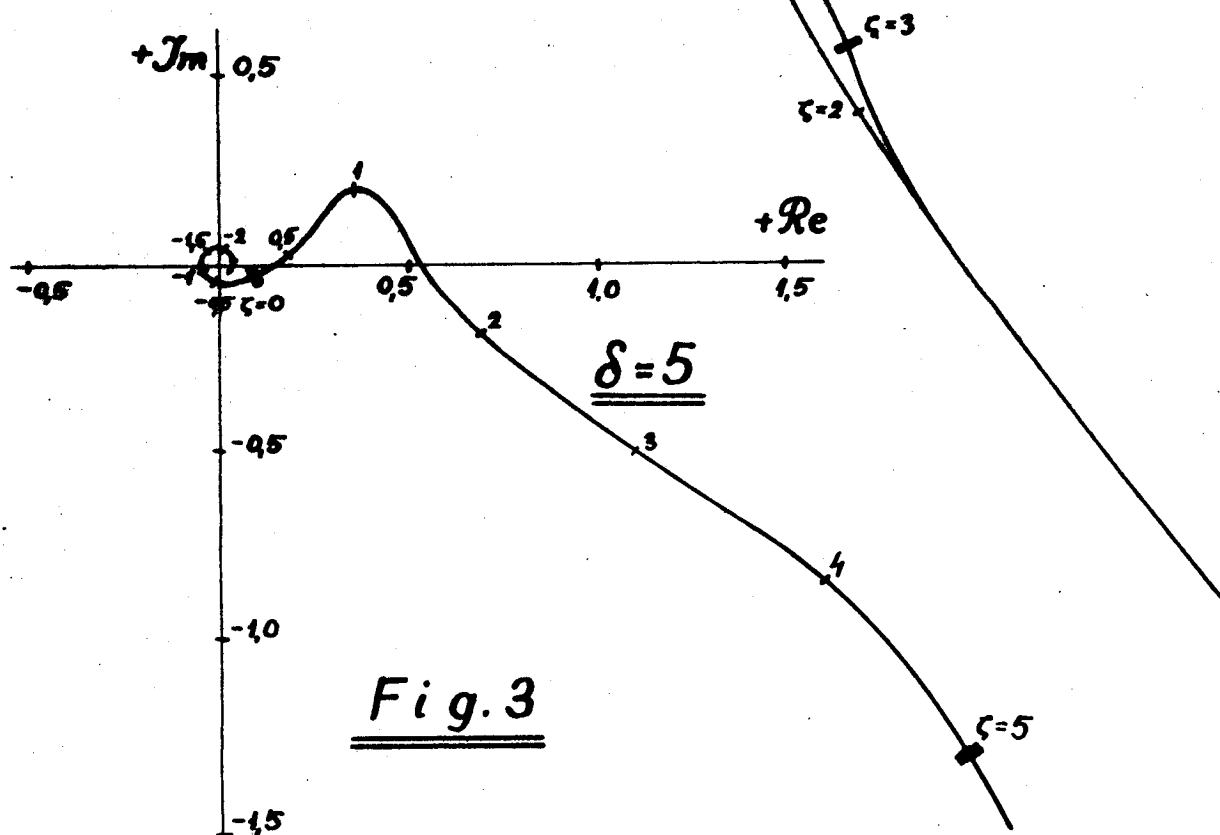
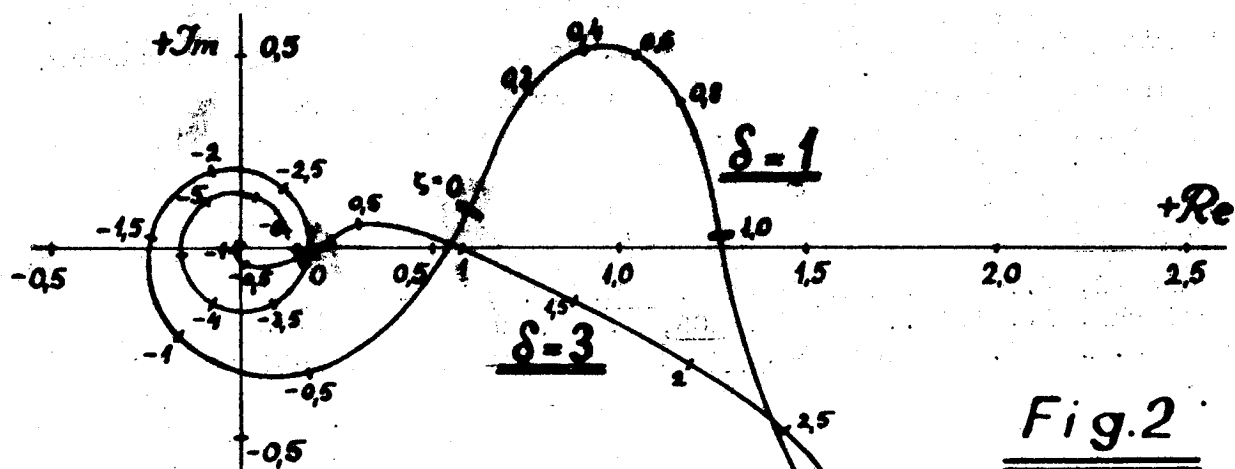
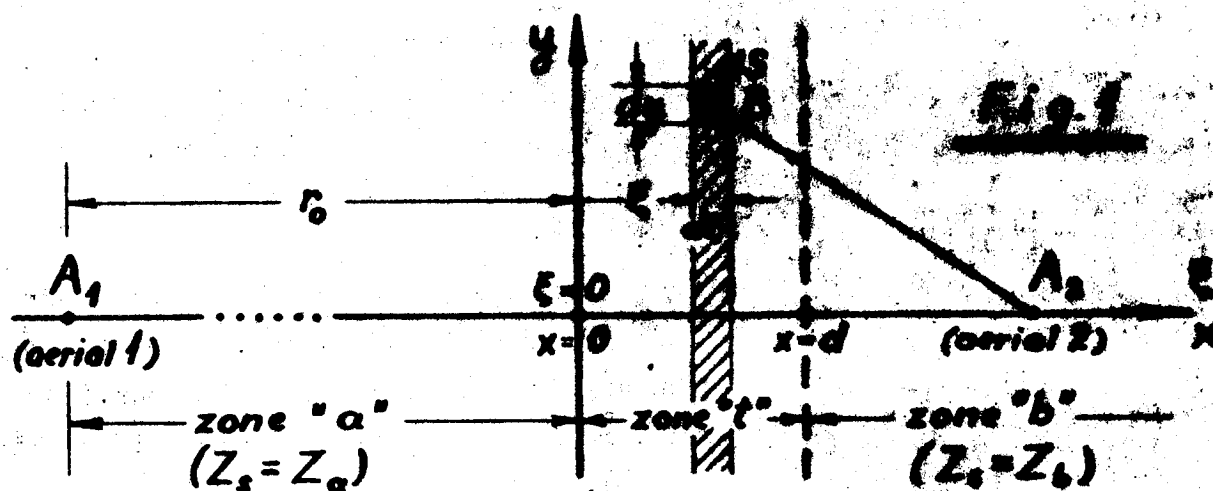
$$\left. \begin{aligned} A_r(u) &= \int e^{-iu} H_1^{(2)}(u) du; & A_t(u) &= \int e^{+iu} H_1^{(2)}(u) du \\ B_r(u) &= \int_0^u u e^{-iu} H_1^{(2)}(u) du; & B_t(u) &= \int_0^u u e^{+iu} H_1^{(2)}(u) du \end{aligned} \right\} \quad (7)$$

Functions  $A_r(u)$ ,  $B_r(u)$  describe the "reflected" fields, i.e. the fields radiated by the secondary sources in the backward direction; they are of oscillatory character. Functions  $A_t(u)$ ,  $B_t(u)$  describe the "transmitted" fields, i.e. the fields radiated in the forward direction; they increase monotonically with  $u$ .

The function  $W(\zeta)$  is multiplied in eqn.(3) by a factor  $-\frac{1}{2} \beta_b$ . This factor is large when the conductivity contrast is pronounced; its largest value it attains for land-sea boundary. Numerically this factor is rather small; it practically never exceeds a few tenths.

Examples of  $W(\zeta) = f(\zeta)$  for  $\delta =$  parameter are given in Figs. 2 and 3. The values of  $\zeta$  are inscribed in the curves and the ends of transition zones are indicated by heavy marks.

To the left of the boundary (for  $\zeta < 0$ ) the field shows oscillations, contrary to conventional theories which predict there the existence of the undisturbed field. As follows from Figs. 2 and 3 and the previous discussion the magnitude of the oscillations is rather small, not exceeding a few per cent; the oscillations are limited to the immediate neighbourhood of the boundary. The magnitude of the oscillations depends on  $\delta = 2\pi d / \lambda_0$ , i.e. the ratio of the width of the transition zone to the free-space wavelength. The oscillations are largest for small  $\delta$ . For  $\delta \approx \pi$  the oscillations are very



small (there occurs "matching" of the zone "a" to the zone "b"). With the increasing width of the transition zone the oscillations increase again showing, however, a magnitude much smaller than before, then decrease to very small values, increase again but also this time to a value smaller than before, etc.

To the right of the boundary (for  $\zeta > \delta$ ) there are no oscillations and the field changes monotonically with distance. For  $\zeta = 2\pi x/\lambda_0 \gg 1$ , i.e. for distances from the boundary much larger than  $\lambda_0/2\pi$  eqn.(3) gives results equivalent to that predicted by the conventional theories. (see for example (Godzinski, 1958)).

Summing up the discussion we may conclude that the conventional theories give in general correct results except very near the boundary (within the distances of the order of wavelength) where small fields disturbances are produced.

#### References

- Godzinski, Z., Proc. IEE, Part C, 105, No.8, 448-464 (September 1958)  
 Godzinski, Z., ibid., Part C, 108, No. 14, 362-373 (September 1961)  
 Wait, J.R., J.Research NBS, 58, No. 5, 237-242 (May 1957)